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TWO-PHASE BOUNDARY-LAYER TREATMENT OF FORCED-CONVECTION FILM BOILING

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Abstract—Theoretical studies have been made of saturated and surface film boiling from a horizontal isothermal plate in a longitudinal flow field. The analysis has been based on the recently developed two-phase boundary-layer theory and the heat transfer and skin friction characteristics are found to be correlated by five parameters. The differential equations have been solved numerically on an electronic digital computer for the parameter range of practical interest. The results are summed up in diagrams, from which the strong dependence of heat transfer and skin friction on the so-called $\rho\mu$-ratio and the rather weak dependence on superheating can be seen.

The present calculations always give larger values of heat-transfer coefficient and drag coefficient than were given by Cess and Sparrow, assuming linear distributions of temperature and velocity component in the longitudinal direction in the vapor film. The parameter range in which Cess and Sparrow's results coincide with ours to within five per cent is shown as a table for saturated film boiling.

NOMENCLATURE

a, thermal diffusivity;
c_p, specific heat;
f, dimensionless velocity function, equations (11) and (12);
g, acceleration due to gravity;
l, latent heat of vaporization;
q, heat flux on heating surface;
u, velocity component in x-direction;
v, velocity component in y-direction;
w, mass flow rate;
x, y, co-ordinates;
C_D, drag coefficient, equation (31);
M, transformation constant, equations (15) and (16);
N, transformation constant, equations (15) and (16);
Nu, Nusselt number, equation (29);
$\overline{Nu}$, average Nusselt number, equation $(\overline{29})$;
Pr, Prandtl number;
R, $\rho\mu$ ratio, equation (23′);
Re, Reynolds number, equation (29);
Sc, dimensionless degree of subcooling, equation (26′);
Sp, dimensionless degree of superheating, equation (26′);
Sp_1, dimensionless constant, equation (26′);
Sp_2, dimensionless constant, equation (26′);
T, temperature;
ΔT_V, degree of superheating, $T_w - T_s$;
ΔT_L, degree of subcooling, $T_s - T_\infty$.

Greek symbols
δ, thickness of vapor film;
η, similarity variable, equations (7) and (8);
θ, dimensionless temperature, equations (13) and (14);
λ, thermal conductivity;
μ, absolute viscosity;
ν, kinematic viscosity;
τ, shearing stress;
ϕ, modified Nusselt number, equation (29);
$\bar{\phi}$, ϕ for average Nusselt number, equation (29);
χ, shearing stress on the wall surface made dimensionless, equation (30);
$\bar{\chi}$, χ for average shearing stress, equation (30);
Ψ, stream function, equations (9) and (10).

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Suffixes

- V, vapor;
- L, liquid;
- i, vapor–liquid interface;
- s, for saturated condition;
- ∞, for condition $y \to \infty$.

1. INTRODUCTION

IT IS SOME years ago that the application of the boundary-layer concept, which has long been known, to the analysis of film condensation or stable film boiling was initiated. The first workers to give the theory of film condensation proposed by Nusselt a drastic change were Sparrow and Gregg [1].

Later on, the papers analysing theseph enomena with the boundary-layer theory appeared one after another [2–7]; the treatments will be classified into two categories, as were many of the analyses of single-phase boundary layer. One is characterized by the procedure of determining the unknown constants contained in the velocity and temperature distribution and the thickness of the boundary layer, the functional forms of which have been chosen beforehand, so that they satisfy the fundamental differential equations integrated across the boundary layer and the boundary conditions. This is the so-called "profile" method. The other is the "similarity transformation" method, in which, by assuming the existence of a stream function and the possibility of similar solutions, one determines similar velocity and temperature distributions so that they satisfy the transformed differential equations and boundary conditions. Depending upon the former, we can often easily evaluate the relations among the dimensionless quantities required and thereby investigate the relations between the dimensionless quantities with good prospect of success. In that case, however, the success or failure of the analysis depends directly upon the question whether the profiles assumed initially express the real ones with sufficient accuracy. On the other hand, while the latter enables us to determine the distribution of velocity and temperature precisely, as far as the similarity relations are possible, the mutual relations of dimensionless quantities can be determined only as diagrams in many cases. This is because the fundamental differential equations can usually only be integrated numerically.

In this paper, a theoretical analysis of forced-convection film boiling from a flat plate has been carried out by means of the method of similarity transformation mentioned above, and the nature of forced-convection surface film boiling has been made clear over a wide range, relaxing the restriction in the range of parameters imposed in Cess and Sparrow's solution [8], which is the only one concerning a similar problem known to the authors.

2. ANALYSIS

2.1. *Physical model*

As shown in Fig. 1, the case studied will be a plate kept at a constant temperature T_w in a liquid flowing at velocity u_∞ at temperature $T_\infty(T_w > T_s > T_\infty$, T_s: saturation temperature corresponding to the system pressure). It will be

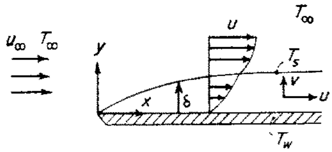

FIG. 1. Physical model and co-ordinates.

assumed that a smooth vapor film of thickness δ is formed along the plate, and that the physical properties of vapor and liquid are constant. Radiative heat transfer will not be considered in this case.

2.2. *Fundamental equations and similarity transformation*

The conservation laws for mass-, momentum- and energy-transfer in the vapor film will be as follows, if the film formed on the heating surface is assumed to constitute a boundary layer:

$$\frac{\partial u_V}{\partial x} + \frac{\partial v_V}{\partial y} = 0 \tag{1}$$

$$u_V \frac{\partial u_V}{\partial x} + v_V \frac{\partial u_V}{\partial y} = \nu_V \frac{\partial^2 u_V}{\partial y^2} \tag{2}$$

$$u_V \frac{\partial T_V}{\partial x} + v_V \frac{\partial T_V}{\partial y} = a_V \frac{\partial^2 T_V}{\partial y^2} \tag{3}$$

It is assumed, however, that there is no pressure gradient in the x-direction, that is, it will be assumed that $\partial u_\infty/\partial x = 0$. Considering next that the liquid near the vapor–liquid interface has the character of a boundary layer, we can obtain similar equations as follows:

$$\frac{\partial u_L}{\partial x} + \frac{\partial v_L}{\partial y} = 0 \tag{4}$$

$$u_L \frac{\partial u_L}{\partial x} + v_L \frac{\partial u_L}{\partial y} = \nu_L \frac{\partial^2 u_L}{\partial y^2} \tag{5}$$

$$u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial y} = a_L \frac{\partial^2 T_L}{\partial y^2} \tag{6}$$

Next comes the similarity transformation following the method of Blasius–Pohlhausen, but here only a simple explanation will be given of the introduction of stream functions and the definition of transformations. First, similarity variables η_V, η_L will be defined as follows including unknown constants N_V, N_L:

$$\eta_V = N_V x^{-\frac{1}{2}} y \tag{7}$$

$$\eta_L = N_L x^{-\frac{1}{2}} y \tag{8}$$

In the second place, by introducing stream functions, the stream functions themselves and the temperature, including unknown constants M_V, M_L, will be expressed further as follows:

$$u_V = \frac{\partial \Psi_V}{\partial y}, \quad v_V = -\frac{\partial \Psi_V}{\partial x} \tag{9}$$

$$u_L = \frac{\partial \Psi_L}{\partial y}, \quad v_L = -\frac{\partial \Psi_L}{\partial x} \tag{10}$$

$$f_V(\eta_V) = \frac{\Psi_V}{M_V x^{\frac{1}{2}}} \tag{11}$$

$$f_L(\eta_D) = \frac{\Psi_L}{M_L x^{\frac{1}{2}}} \tag{12}$$

$$\theta_V(\eta_V) = \frac{T_V - T_s}{T_w - T_s} = \frac{T_V - T_s}{\Delta T_V} \tag{13}$$

$$\theta_L(\eta_L) = \frac{T_L - T_\infty}{T_s - T_\infty} = \frac{T_L - T_\infty}{\Delta T_L} \tag{14}$$

Assuming that f_V, f_L, θ_V and θ_L are functions only of η_V or η_L, and substituting these transformation relations in the fundamental equations (1)–(6), and determining the constants M_V, M_L, N_V and N_L as follows:

$$M_V = \sqrt{(u_\infty \nu_V)}, \quad N_V = (1/2)\sqrt{(u_\infty/\nu_V)} \tag{15}$$

$$M_L = \sqrt{(u_\infty \nu_L)}, \quad N_L = (1/2)\sqrt{(u_\infty/\nu_L)} \tag{16}$$

the fundamental equations (1)–(6) will be replaced by the following four differential equations:

$$f_V''' + f_V f_V'' = 0 \tag{17}$$

$$\theta_V'' + Pr_V f_V \theta_V' = 0 \tag{18}$$

$$f_L''' + f_L f_L'' = 0 \tag{19}$$

$$\theta_L'' + Pr_L f_L \theta_L' = 0 \tag{20}$$

Here the primes represent differentiation with respect to η_V in the vapor phase and η_L in the liquid phase respectively.

2.3. *Boundary- and matching-conditions*

The conditions on the heating surface will be as follows, as in the case of the single phase.

$$y = 0: \quad u_V = v_V = 0, \quad T_V' = T_w \tag{21}$$

$$\eta_V = 0: \quad f_V = f_V' = 0, \quad \theta_V = 1 \tag{21'}$$

Now the profile of the vapor–liquid interface is to be determined in relation to the vapor-film thickness δ by the equation

$$(\eta_V)_i = (\delta/2)\sqrt{(u_\infty/(\nu_V x))}.$$

Since the dimensionless vapor-film thickness cannot be changed in the x-direction in order to carry on the analysis by assuming a similar solution, the only possible development of the vapor film must be of the $\delta \propto \sqrt{x}$ type.

While it is necessary to describe the continuity and compatibility condition of the various physical quantities on such a vapor–liquid interface, at this point the theory of the two-phase boundary layer is in marked contrast to that of the single phase, and the accuracy of the solution and the difficulty in getting a solution largely depend upon the treatment at this point. Efforts have been made here to express them as precisely as possible within the concept of the boundary layer.

First, the conditions relating to temperature will be written simply as follows

$$y = \delta: \quad T_V = T_L = T_s \tag{22}$$

$$\eta_V = (\eta_V)_i = \eta_L: \quad \theta_V = 0, \quad \theta_L = 1 \tag{22'}$$

In (22′) we have $\eta_V = \eta_L$ at the vapor–liquid interface. It ought to be expressed correctly as $\eta_L = (\eta_V)_i\sqrt{(\nu_V/\nu_L)}$, but equation (22′) will serve the purpose conveniently, since η_V and η_L appear neither in the differential equations nor in the boundary- and matching-conditions.

Now the first condition, concerning the velocity component, can be obtained from the consideration that the mass transfer across the vapor–liquid interface is continuous:

$$y = \delta: \quad \rho_V\left(v_V - u_V\frac{d\delta}{dx}\right)_i = \rho_L\left(v_L - u_L\frac{d\delta}{dx}\right)_i \tag{23}$$

$$\eta_V = (\eta_V)_i = \eta_L: \quad (f_L)_i = R(f_V)_i, \quad R \equiv \sqrt{\left(\frac{(\rho\mu)_V}{(\rho\mu)_L}\right)} \tag{23'}$$

Assuming next the continuity of the velocity component in a tangential direction at the vapor–liquid interface, we can introduce the second condition concerning the velocity component, which can be calculated with good accuracy as follows, according to the boundary-layer treatment:

$$y = \delta: \quad (u_V)_i = (u_L)_i \tag{24}$$

$$\eta_V = (\eta_V)_i = \eta_L: \quad (f_L')_i = (f_V')_i \tag{24'}$$

Furthermore, as the third condition, we will impose the compatibility condition of shearing stress upon the vapor–liquid interface. In this case also, we can express the condition with sufficient precision as follows, by the boundary-layer treatment:

$$y = \delta: \quad \mu_V\left(\frac{\partial u_V}{\partial y}\right)_i = \mu_L\left(\frac{\partial u_L}{\partial y}\right)_i \tag{25}$$

$$\eta_V = (\eta_V)_i = \eta_L: \quad (f_L'')_i = (f_V'')_i \tag{25'}$$

Incidentally, the heat balance at the vapor–liquid interface is written as follows, though it is not intended to use it as a boundary condition:

$$\left.\begin{aligned} &y = \delta: \\ &\left(-\lambda_V\frac{\partial T_V}{\partial y}\right)_i = -(w)_i l + \left(-\lambda_L\frac{\partial T_L}{\partial y}\right)_i \\ &(w)_i = \rho_V\left(v_V - u_V\frac{d\delta}{dx}\right)_i \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} &\eta_V = (\eta_V)_i = \eta_L: \\ &\quad Sp = Sp_1 + Sp_2 Sc \\ &\quad Sp \equiv \frac{c_{p_V}\Delta T_V}{Pr_V l R^2}, \quad Sc \equiv \frac{c_{p_V}\Delta T_L}{\sqrt{(Pr_L)}\, l R^2} \\ &\quad Sp_1 \equiv \frac{-(f_V)_i}{R^2(\theta_V')_i}, \quad Sp_2 \equiv \frac{(\theta_L')_i}{\sqrt{(Pr_L)}\, R(\theta_V')_i} \end{aligned}\right\} \tag{26'}$$

These equations give the relation between the degree of superheating and subcooling. Note that the parameter Γ used in Cess and Sparrow's analysis can be expressed as $\sqrt{(Pr)_L}\,\Gamma = Sc/Sp$ in the present nomenclature.

On the other hand, the conditions for $y \to \infty$ are as follows:

$$y \to \infty: \quad u_L \to u_\infty \tag{27}$$

$$\eta_L \to \infty: \quad f_L' \to 2 \tag{27'}$$

$$y \to \infty: \quad T_L \to T_\infty \tag{28}$$

$$\eta_L \to \infty: \quad \theta_L \to 0 \tag{28'}$$

2.4. *Heat transfer and skin friction*

The characteristics of heat transfer will be expressed by the following equations:

$$\left.\begin{aligned} &\phi \equiv \frac{Nu}{\sqrt{(Re)}}\left(\frac{\mu_V}{\mu_L}\right) = -\frac{(\theta_V')_w}{2}\cdot R, \\ &\bar{\phi} \equiv \frac{\overline{Nu}}{\sqrt{(Re)}}\left(\frac{\mu_V}{\mu_L}\right) = -(\theta_V')_w R = 2\phi \\ &Nu \equiv \frac{qx}{\lambda_V \Delta T_V}, \quad \overline{Nu} \equiv \frac{\bar{q}x}{\lambda_V \Delta T_V}, \\ &q = -\lambda_V\left(\frac{\partial T_V}{\partial y}\right)_w, \\ &\bar{q} = \frac{1}{x}\int_0^x q\,dx, \quad Re \equiv \frac{u_\infty x}{\nu_L} \end{aligned}\right\} \tag{29}$$

The dimensionless expression for the shearing stress acting on the heating surface can be written as follows:

$$\left.\begin{aligned} \chi &\equiv \frac{\tau_w}{\rho_L u_\infty^2}\sqrt{(Re)} = \frac{(f_V'')_w}{4}\cdot R, \\ \bar{\chi} &\equiv \frac{\overline{\tau_w}}{\rho_L u_\infty^2}\sqrt{(Re)} = \frac{(f_V'')_w}{2} R = 2\chi \\ \tau_w &= \mu_V\left(\frac{\partial u_V}{\partial y}\right)_w, \quad \overline{\tau_w} = \frac{1}{x}\int_0^x \tau_w \, dx \end{aligned}\right\} \quad (30)$$

Therefore, the expression in the form of a drag coefficient will be as follows:

$$C_D\sqrt{(Re)} = (f_V'')_w \cdot R = 4\chi,$$

$$C_D \equiv \frac{\int_0^x \tau_w \, dx}{\frac{\rho_L u_\infty^2}{2}\cdot x} \quad (31)$$

2.5. *Numerical calculations*

Now we have all the equations required. What remains to be done as a problem of numerical analysis is to evaluate solutions which satisfy the transformed fundamental differential equations (17)–(20) and the boundary- and matching-conditions (21′)–(28′).

No term concerning the temperature is included in the momentum equations (17) and (19). Dimensionless velocity functions f_V and f_L will be therefore determined uniquely* by giving the $\rho\mu$ ratio R related to the compatibility of the velocity component at the vapor–liquid interface, and the dimensionless vapor film thickness $(\eta_V)_i$. Accordingly, the velocity distribution for specified R and $(\eta_V)_i$ will be determined first. Next, the distribution of dimensionless temperature will be evaluated, using the velocity distribution just evaluated, and using integrations of equations (18) and (20) for the combination of optional Prandtl numbers:

$$\theta_V = \frac{\int_{\eta_V}^{(\eta_V)_i} \exp\left(-Pr_V \int_0^{\eta_V} f_V \, d\eta_V\right) d\eta_V}{\int_0^{(\eta_V)_i} \exp\left(-Pr_V \int_0^{\eta_V} f_V \, d\eta_V\right) d\eta_V} \quad (18')$$

$$\theta_L = \frac{\int_{\eta_L}^{\infty} \exp\left(-Pr_L \int_{(\eta_V)_i}^{\eta_L} f_L \, d\eta_L\right) d\eta_L}{\int_{(\eta_V)_i}^{\infty} \exp\left(-Pr_L \int_{(\eta_V)_i}^{\eta_L} f_L \, d\eta_L\right) d\eta_L} \quad (20')$$

Since Sp_1 and Sp_2 are determined by the equation (26′) when the dimensionless velocity and temperature have been thus evaluated, the relation between the degree of superheating and subcooling is made clear. Furthermore, the value of the heat transfer is determined by equation (29) and that of the skin friction by equation (30) or (31).

When the foregoing process is repeated for a fixed $\rho\mu$ ratio R, for different values of the dimensionless vapor film thickness $(\eta_V)_i$, the modified Nusselt number ϕ and the dimensionless skin friction χ will be expressed by the following forms:

$$\phi = \phi(Pr_V, Pr_L, R, Sc, Sp) \quad (32)$$

$$\chi = \chi(Pr_V, Pr_L, R, Sc, Sp) \quad (33)$$

The range of parameters covered by the numerical calculations is as follows:

R: 0·005, 0·01, 0·02, 0·04, 0·08, 0·16, 0·32, 0·64

Pr_V: 0·5, 1·0, 2·0; Pr_L: 0·5, 1·0, 2·0, 4·0, 8·0

These may be considered to cover the domain of the parameters for the real fluid at the usual pressures.

3. RESULTS AND CONSIDERATIONS

The solutions of the form (32) for heat transfer are shown in Fig. 2(a), (b), (c), (d) and (e), and the solutions of the form (33) for skin friction are shown in Fig. 3.

Though only values for $Pr_L = 2{\cdot}0$ are given in Fig. 2, they can be used in practice since the value of the ordinate changes no more than $\pm 5\%$ around the value for $Pr_L = 2{\cdot}0$ within the range of $0{\cdot}5 \leqq Pr_L \leqq 8{\cdot}0$, provided $R \leqq 0{\cdot}16$. If, however, R increases as much as 0·64, the

* The differential equations (17) and (19) can be integrated as an initial value problem instead of a boundary value problem; see, for example, reference 9.

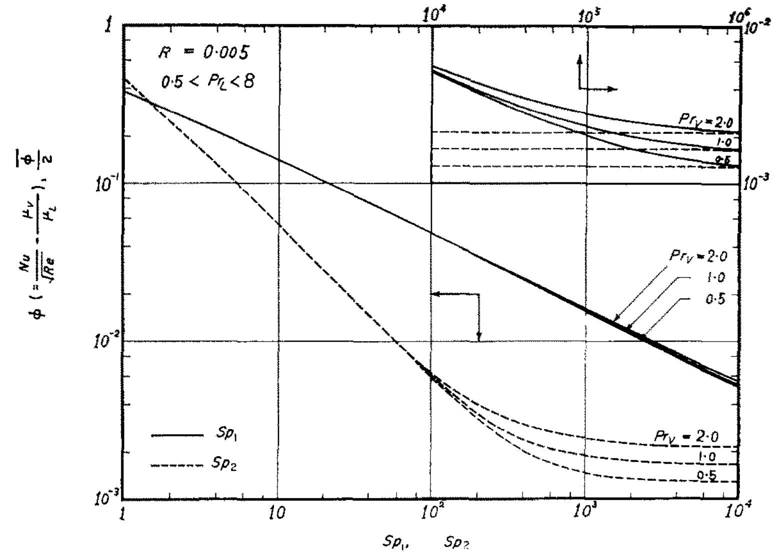

FIG. 2(a). Correlation of heat transfer at $R = 0{\cdot}005$.

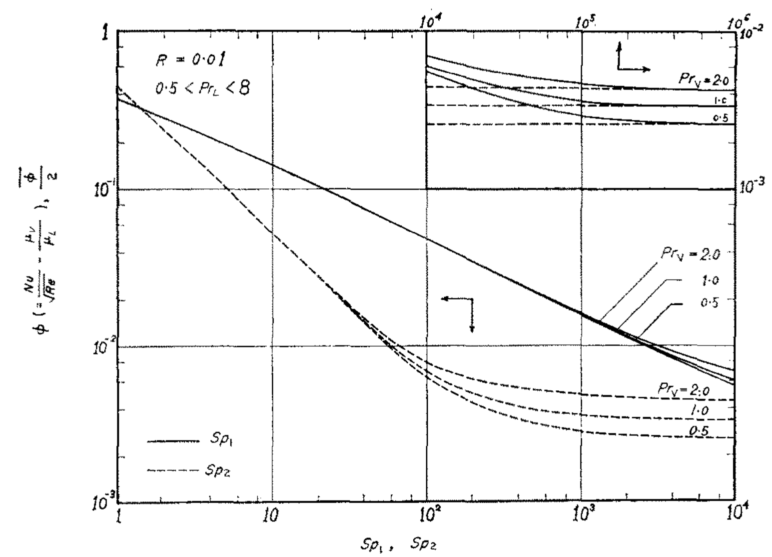

FIG. 2(b). Correlation of heat transfer at $R = 0{\cdot}01$.

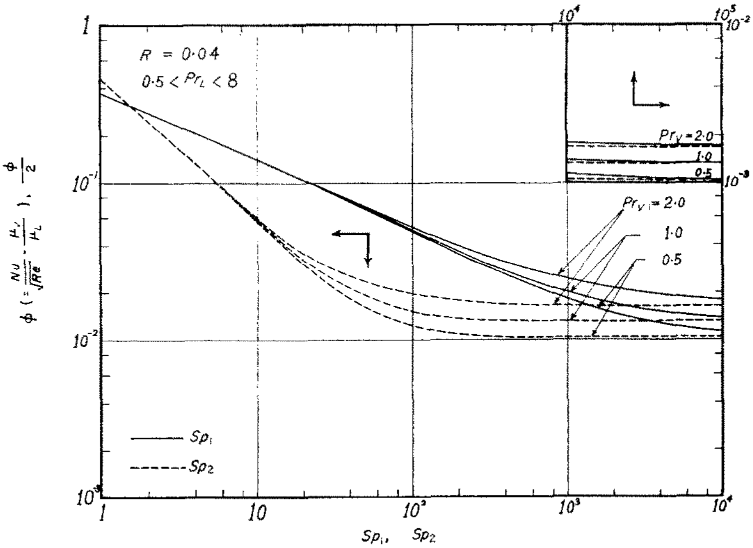

FIG. 2(c). Correlation of heat transfer at $R = 0{\cdot}04$.

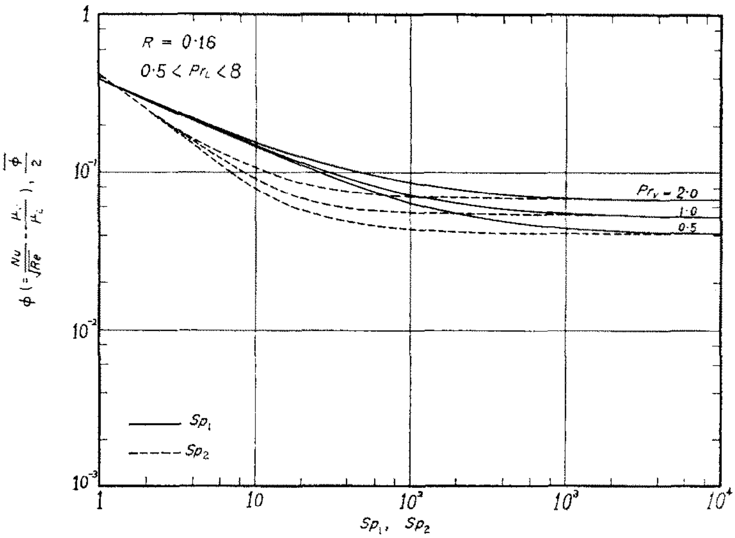

FIG. 2(d). Correlation of heat transfer at $R = 0{\cdot}16$.

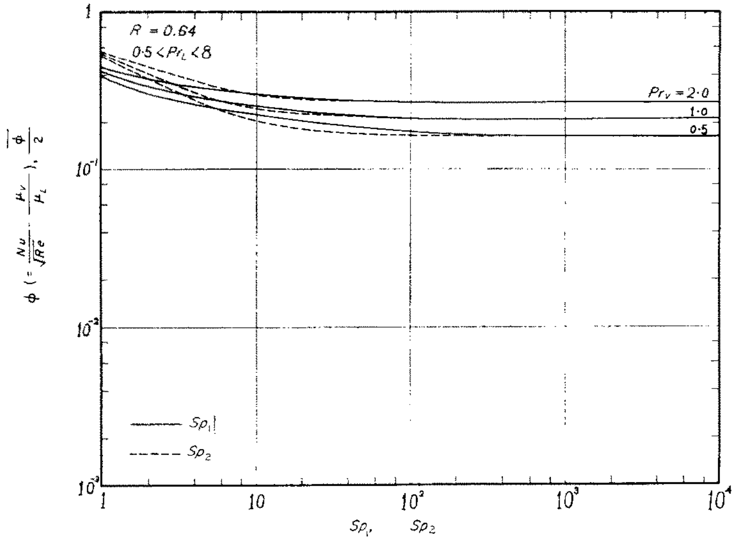

FIG. 2(e). Correlation of heat transfer at $R = 0{\cdot}64$.

value of ordinate changes by $\pm 10\%$. On the other hand, as the values of χ change by $\pm 10\%$ at most within the range of $0{\cdot}5 \leqq Pr_V \leqq 2{\cdot}0$ when $R \leqq 0{\cdot}64$, only the results for $Pr_V = 1{\cdot}0$ are shown in Fig. 3.

3.1. *How to read the diagrams*

Since we have $Sc = 0$ in the equation (26′) when there is no degree of subcooling, that is, when the liquid is uniformly at saturation temperature, we can read at once the relations of ϕ, $\bar{\phi} \sim Sp$ and χ, $\bar{\chi}$, $C_D \sim Sp$ for given R and Pr_V.

Next in order to evaluate ϕ or χ in a case where there is a degree of subcooling, and where Sp and Sc are given beside R and Pr_V, the following method is advisable, though a little troublesome. For example, when $R = 0{\cdot}005$, $Pr_V = 1{\cdot}0$, ($Pr_L = 2{\cdot}0$), $Sp = 22\,000$ and $Sc = 530$, the curve for $Pr_V = 1{\cdot}0$ in Fig. 2(a) and that for $R = 0{\cdot}005$ in Fig. 3 are used.

(i) Assuming Sp_1 in Fig. 2 (a), we can read ϕ and Sp_2 for the value of prescribed Pr_V (1.0).

(ii) From the values of Sp_1, Sp_2 and the prescribed value of Sc (530), Sp is calculated using the equation (26′).

(iii) If Sp that was evaluated in (ii) does not agree with the given Sp (22 000), revert again to (i).

(iv) For the value of Sp_1 thus determined χ will be read from Fig. 3 for the specified value of R(0·005).

Thus the evaluation of $Sp_1 = 1150$, $Sp_2 = 39$, $\phi = 0{\cdot}015$, $\chi = 0{\cdot}014$ can be done. This example almost corresponds to the case of water at atmospheric pressure with superheating of 700°C and subcooling of 10°C.

Even in the case where there is a degree of subcooling, evaluation is simple when ϕ or χ is given beside R and Pr_V. When ϕ is given, we can read Sp_1 and Sp_2 from Fig. 2. Incidentally, for the Sp_1 thus obtained, χ can be read from Fig. 3. When χ is given, Sp_1 is read first from Fig. 3 and for that Sp_1 we can read ϕ and Sp_2 from Fig. 2. When Sc is given optionally for Sp_1 and Sp_2 evaluated in these cases, Sp can be determined from the equation (26′).

3.2. *Comparison with Cess and Sparrow's solution* [8]

Starting from the two assumptions, (1) in the vapor phase, the temperature and velocity components in x direction, T_V and u_V, vary linearly

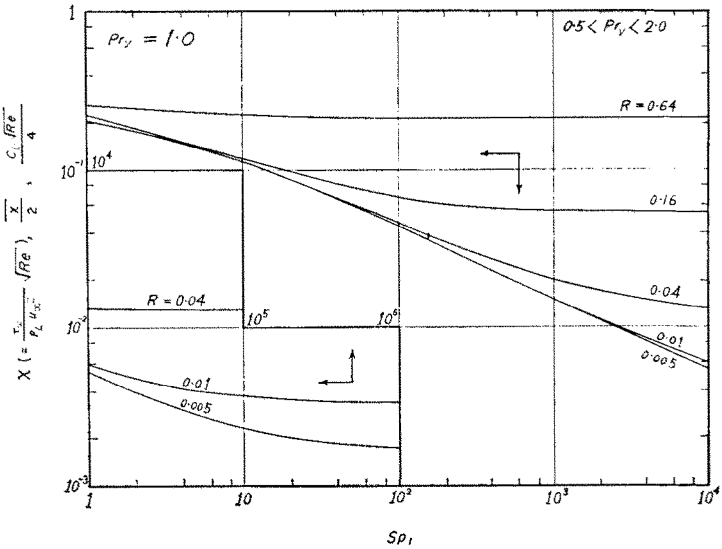

FIG. 3. Correlation of skin friction.

with distance from the heating surface y, and (2) $(f_L)_i = 0$ approximately, Cess and Sparrow have proposed approximate solutions by a combined analytical–numerical method for the case where there is no degree of subcooling. They have further sought approximate solutions for the subcooled liquid by enlarging the solutions for the saturated liquid, which are quoted in Figs. 4 and 5 for a liquid Prandtl number of unity. The broken lines in the figure are the results from the present analysis for saturated liquid and vapor Prandtl number of unity. The values of heat transfer and skin friction show considerable differences in comparison with the solid lines for $\Gamma\sqrt{(Pr_L)} = 0$, especially for large R.

To know when such assumptions as (1) and (2) are permissible and to find out how to express the limit of parameters in which the solutions obtained by using them are reasonable, the authors will investigate first the case of saturated liquid and then that of subcooled liquid.

When there exists no degree of subcooling, the only difference between the theory of the present paper and that of Cess and Sparrow is that the former does not require the assumptions (1) and (2). All we have to do, therefore, is to find out the limit of parameters in which assumptions (1) and (2) are allowed.

As a preliminary, we must obtain series solutions for the transformed fundamental differential equations (17) and (18) of the vapor side. With such solutions in hand, the assumption (1) by which velocity and temperature are supposed to vary linearly is considered appropriate only when such a condition, as rendering the terms of lowest power giving non-linear distributions negligibly small compared with the terms of highest power giving linear distributions is satisfied as discussed in Cess and Sparrow's paper. When calculated it will be expressed when Prandtl number of vapor Pr_V is about unity as follows:

$$Sp \equiv \frac{c_{pV}\Delta T_V}{Pr_V l R^2} \ll \frac{12}{R^2}$$

In other words, the assumption is permissible only when the dimensionless degree of superheating Sp is smaller than $12/R^2$ which depends exclusively on the magnitude of the $\rho\mu$ ratio R, the values of which are given in Figs. 4 and 5 for each R with vertical markers. It is added, however, that the values for $R = 0.01$ and 0.005 are 1.2×10^5 and 4.8×10^5 respectively. This limiting value is inversely proportional to Pr_V when not unity.

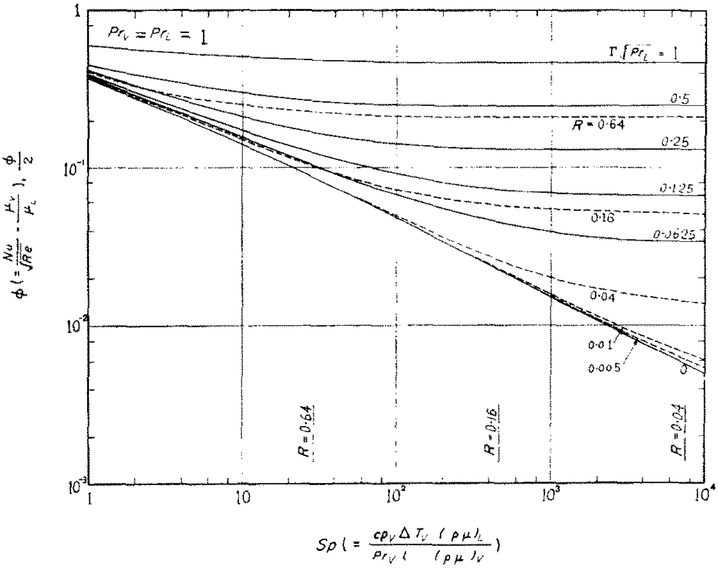

FIG. 4. Correlation of heat transfer proposed by Cess and Sparrow.

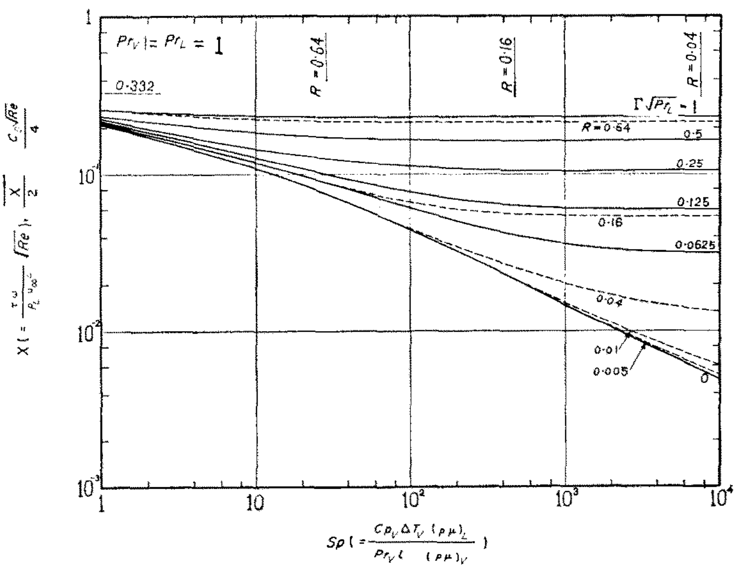

FIG. 5. Correlation of skin friction proposed by Cess and Sparrow.

The assumption $(f_L)_i \cong 0$ is valid only when the $\rho\mu$ ratio R and the dimensionless degree of superheating Sp are sufficiently small. Cess and Sparrow compared the solutions based on this assumption with those from which this assumption is excluded giving the value as large as 0·6 about R, and concluded that the error for the magnitudes of ϕ and χ originating from the assumption is not higher than -15%. The consideration here however is limited to the case in which the dimensionless degree of superheating Sp is as small as unity. Therefore the range of parameters where Cess and Sparrow's solutions are reasonable is restricted also from the assumption (2).

Figure 6 shows how the relation among $(f_L)_i$, $(f_L')_i$ and $(f_L'')_i$ changes with respect to the $\rho\mu$ ratio R. It is clearly seen from the figure that, since a large value of the abscissa $(f_L')_i$ implies a large degree of superheating Sp, $(f_L)_i \cong 0$ or assumption (2) ceases to be satisfied with the increase of Sp and, needless to say, the $\rho\mu$ ratio R.

On the basis of the foregoing discussions on the assumptions (1) and (2), the authors will investigate the temperature and velocity distributions, as shown in Fig. 7, evaluated by the method of the

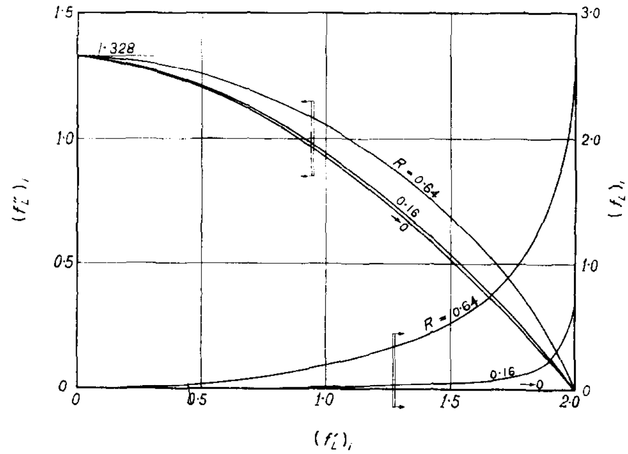

FIG. 6. Relation among $(f_L)_i$, $(f'_L)_i$, $(f''_L)_i$ and R.

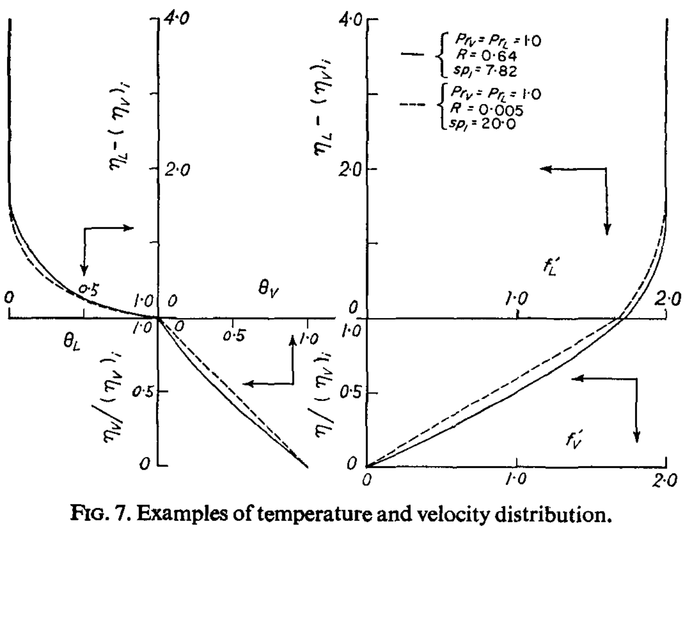

FIG. 7. Examples of temperature and velocity distribution.

present paper in which neither assumption is used. In the case of the curves drawn in a broken line, since the dimensionless degree of superheating is much smaller than the limiting value of $1{\cdot}2 \times 10^5$ stated above, the distributions of temperature and velocity in the vapor film are seen to be extremely linear. In the case of the curves indicated by a solid line, however, the distributions have become considerably nonlinear, despite the fact that the dimensionless degree of superheating is as small as one-third of the limiting value of 29. Again in the first example, the values of $(f_L)_i$ and $(f_L'')_i$ introduced from the assumptions (1) and (2) are, as seen in Table 1, in good agreement with those evaluated by means of the method of the present paper, whereas in the second example there are marked differences.

Table 1. Scrutinization of the assumptions (1) *and* (2) *in the example of Fig.* 7. (*Numbers in parentheses represent values given by the assumptions.*)

	$(f_L')_i$	$(f_L)_i$	$(f_L'')_i$
$Pr_V = Pr_L = 1{\cdot}0$ $R = 0{\cdot}64$ $Sp_1 = 7{\cdot}82$	1·72	0·861 (0)	0·464 (0·301)
$Pr_V = Pr_L = 1{\cdot}0$ $R = 0{\cdot}005$ $Sp_1 = 20{\cdot}0$	1·68	0·000102 (0)	0·344 (0·342)

After all, considering the discussions mentioned above and comparing Cess and Sparrow's solutions with the present results, the authors have reached the conclusion that the dimensionless degree of superheating should not exceed the values shown in Table 2, if the values of ϕ and χ due to the former are to be brought into agreement with those due to the latter within five per cent.

Table 2. Limiting value of the dimensionless degree of superheating for Cess and Sparrow's result coinciding with the present result to within five per cent

Pr_V	R				
	0·005	00·1	0·04	0·16	0·64
0·5	11 000	3800	180	6·8	0·48
1·0	6900	1700	85	5·2	0·48
2·0	1300	840	34	5·0	0·35

Next, consideration will be directed to the case where a degree of subcooling exists. It is not necessary in the analysis of the present paper to make a special distinction in the method of solution whether or not there is a degree of subcooling, but Cess and Sparrow's solutions for the saturated liquid have been enlarged by the same authors as explained below.

First, when the liquid Prandtl number is unity there is a similarity between the temperature and velocity distributions, resulting in the following equation:

$$(\theta_L')_i = -\frac{(f_L'')_i}{2 - (f_L')_i}, \quad Pr_L = 1$$

In such a case, therefore, the solutions for subcooled liquid are derived without difficulty. It is the results thus sought that are shown in Figs. 4 and 5.

Furthermore, when the liquid Prandtl number is either sufficiently small or sufficiently large, the following equations will result:

$$(\theta_L')_i = -2\sqrt{\left(\frac{Pr_L}{\pi}\right)}, \quad Pr_L \ll 1$$

$$(\theta_L')_i = -\sqrt{\left(\frac{2\,Pr_L(f_L')_i}{\pi}\right)}, \quad Pr_L \gg 1$$

Accordingly, the solutions for saturated liquid can be easily converted into those for subcooled liquid.

The Cess and Sparrow solutions for subcooled liquids are, therefore applicable as long as the liquid Prandtl number satisfies one of the above conditions, and provided that the range of parameters is within that for which the solutions for saturated liquid are reasonable.

Now checking our results, we find that the effect of the liquid Prandtl number is not so strong, if the value of the $\rho\mu$ ratio R is not larger than 0·16, as stated in the beginning of the present section; by varying the liquid Prandtl number in the range $0{\cdot}5 \leqq Pr_L \leqq 8{\cdot}0$ the values of ϕ change around that for $Pr_L = 2{\cdot}0$ by only $\pm 5\%$ at most for $Sp_2 \geqq 1$ (the same is true for the values of χ). It is therefore considered that the effect of the Prandtl number of the liquid for a pressure level which is not very high is negligible for normal fluids. Also with Cess and Sparrow's result for subcooled liquid, the calculations for $Pr_L = 1{\cdot}0$

may be adapted to all other cases if the condition imposed on the dimensionless degree of superheating when there is no subcooling is taken into consideration.

What has been discussed so far will show that the Cess and Sparrow results are quite reasonable when the degree of superheating is not too large, whether or not there is any subcooling, and in such a case, they are not only simpler than those of the present paper but estimate accurately enough heat transfer as well as skin friction. In the case of large superheating, however, the error will become large, since the basic assumptions lose their validity. The main point in the authors' results lies in that this restriction upon the degree of superheating is lightened to an extent which makes the application of the boundary-layer theory appropriate. With the decline of the degree of superheating, therefore, the authors' results gradually approach those of Cess and Sparrow.

At the present stage, however, where we have yet to discover experimental values or the extent of the parameter domain which is important for practical use, it is of course unreasonable to discuss the significance of these two analyses from the analytical viewpoint only.

3.3. *Decrease of skin friction due to two-phase flow*

The problem of how the drag on a plate in a flow of liquid changes when a vapor film has been artificially made to attach to its surface, is very interesting practically. In the present case where there is a film of vapor generated from the flowing liquid, as seen from equations (25′) and (31), the drag coefficient becomes as follows with gradual thickening of vapor film:

$$C_D\sqrt{(Re)} \to 1{\cdot}328 \times R$$

and in an extremely thinned-out vapor film,

$$C_D\sqrt{(Re)} \to 1{\cdot}328$$

it approaches the value for a single phase, that is, that of the case where only liquid is flowing. If the $\rho\mu$ ratio is very small and the degree of superheating large, the decrease of the drag force accompanying the formation of a vapor film is considerable.

4. CONCLUSIONS

An analysis of forced convection film boiling has been carried out using the concept of the two-phase boundary layer, and solutions applicable to an extensive range of parameters have been obtained. In applying the results obtained in the present paper to actual problems, the following conditions should be satisfied: (1) the vapor–liquid interface must be smooth, (2) the vapor generated must not leak out into the liquid in the form of bubbles. In order to confirm what effect these conditions have on the results of this analysis, the quickest way is considered to be in the comparison with the experimental results, but for the present, we cannot find any reliable data for such a case.

Machine solutions were obtained on an Okitac 5090 A type electronic computer. It took 12 h to make the actual calculations. Numerical integration of differential equations was carried out by means of the predictor–corrector method.

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Résumé—On a étudié théoriquement l'ébullition saturée et par film superficiel à partir d'une plaque horizontale isotherme dans un écoulement longitudinal. L'analyse a été basée sur la théorie récente de la couche limite diphasique et l'on a trouvé que les caractéristiques de transport de chaleur et de frottement pariétal s'expriment à l'aide de cinq paramètres. Les équations différentielles ont été résolues numériquement sur un calculateur életronique numérique dans la gamme de paramètres d'intérêt pratique. Les résultats sont résumés dans des diagrammes, à partir desquels on peut voir que le transport de chaleur et le frottement pariétal dépend beaucoup du rapport des $\rho\mu$ et faiblement de la surchauffe.

Les calculs actuels donnent toujours pour les coefficients de transport de chaleur et de traînée des valeurs plus élevées que celles données par Cess et Sparrow, qui supposaient des distributions linéaires de température et de composante longitudinale de la vitesse dans le film de vapeur. La gamme de paramètres dans laquelle les résultats de Cess et Sparrow coïncident avec les nôtres à moins de cinq pour cent est donnée sous la forme d'un tableau pour l'ébullition par film saturé.

Zusammenfassung—Theoretische Studien wurden durchgeführt über Sieden mit Dampferzeugung und örtliches Sieden an einer waagerechten isothermen Platte in einem Längsströmungsfeld. Die Analyse beruht auf der kürzlich entwickelten Zweiphasen-Grenzschichttheorie und es zeigt sich, dass Wärmeübergangs- und Reibungscharakteristika durch fünf Parameter korreliert werden können. Die Differentialgleichungen wurden für den praktisch interessierenden Parameterbereich auf einer elektronischen Digitalrechenmaschine numerisch gelöst. Die Ergebnisse sind in Diagrammen zusammengestellt aus welchen die starke Abhängigkeit des Wärmeübergangs und der Oberflächenreibung vom sogenannten $\rho\mu$ Verhältnis und die geringe Abhängigkeit von der Überhitzung erkennbar ist. Die gegenwärtigen Berechnungen ergeben stets grössere Werte für Wärmeübergang und Druckabfall als von Cess und Sparrow angegeben, wobei lineare Temperatur- und Geschwindigkeitskomponentenverteilung in Längsrichtung im Dampffilm angenommen wird. Der Parameterbereich in dem die Ergebnisse von Cess und Sparrow mit unseren innerhalb von 5 Prozent übereinstimmen ist in einer Tabelle für Sieden mit Dampferzeugung gezeigt.

Аннотация—Выполнено теоретическое исследование насыщенного поверхностного пленочного кипения на горизонтальной изотермической пластине в поле продольного течения. Анализ основывался на недавно разработанной теории двухфазного пограничного слоя. Найдено, что характеристики теплообмена и поверхностного трения коррелируются пятью параметрами. На электронной вычислительной машине получены численные решения дифференциальных уравнений для диапазона параметров, представляющего практический интерес. Результаты представлены в виде диаграмм, из которых видно, что на теплообмен и поверхностное трение оказывает большое влияние так называемое отношение $\rho\mu$ и довольно слабое–перегрев.

При таких расчетах коэффициенты теплообмена и коэффициента трения всегда получаются больше значений, приведенных Цессом и Спэрроу, которые принимали линейное распределение температуры и компонента скорости в продольном направлении в пленке пара. Диапазон параметров, в котором результаты Цесса и Спэрроу совпадают с нашими с точностью до 5 процентов, дан в виде таблицы для насыщенного пленочного кипения.